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Experimental evidence of tunneling as a stochastic process

A. Ranfagni,[1] R. Ruggeri,[1] C. Susini,[1] A. Agresti,[2] and P. Sandri[3]
[1]Istituto di Ricerca sulle Onde Eletromagnetiche ''Nello Carrara,'' CNR, Via Panciatichi 64, 50127 Firenze, Italy
[2]Dipartimento di Fisica dell'Università di Firenze, Sezione di Fisica Superiore, Via S. Marta 3, 50139, Firenze, Italy
and Istituto Nazionale di Finica della Materia, Unitá di Ricerca di Firenza, Firenza, Italy
[3]Scuola di Specializzazione in Ottica dell'Università di Firenze, Firenze, Italy

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A model for tunneling based on stochastic processes proves to be capable of interpreting the results of two experiments at the microwave scale. The first of these consisted of measuring the penetration time in a subcutoff waveguide; the second one, in measuring the shift of a beam in a frustrated total reflection. Said shift which is a measurement of the traversal time of the barrier. In both cases, a peak in the real-time component was evidenced, as predicted by the theoretical model.

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Tunneling processes are interesting also in connection with superluminal aspects, even if cases of the absence of dispersion, which is always present in tunneling, give more evident and clearer results [1]. Avoiding for the moment a consideration of these aspects, we shall now reconsider the question of tunneling time in relation to an already-formulated stochastic model [2], in order to demonstrate some interesting new aspects. According to the pioneering work by Kac [3], then developed by DeWitt-Morette and Foong [4] and subsequently applied to tunneling [5], a stochastic model for tunneling processes has been successfully applied in order to interpret experiments with Josephson junctions [6] and optical tunneling in the microwave range [2,7]. However, in all previous experimental demonstrations, the variable ranges were not sufficiently extended so as to obtain a clear demonstration of the validity of the stochastic model. Here, on the contrary, the extension of the field of variables was sufficient to give unambiguous proof of the theoretical predictions. Another aspect is represented by the existence and the detection of a sort of resonance which is peculiar to this kind of approach: based on a stochastic motion of the ''particle'' inside the forbidden region of the barrier, this approach can be also related to highly irregular paths in quantum mechanics predicted by Feynman and Hibbs [8].

The salient results of the stochastic model can be schematized as follows: the average tunneling time is a complex quantity expressed, for L/v tending to zero, as (see the last paragraph)

$$\langle t\rangle \simeq a\left(\frac{L}{v}\right)^2 + i\frac{L}{v}, \qquad (1)$$

where L/v is the semiclassical time that represents the imaginary part, a is the friction coefficient that enters the telegrapher's equation, and is ''responsible'' for the real part, which is zero if $a=0$ and depends quadratically on L/v. However, if a is zero, do we have to believe that a real part of the tunneling time does not exist? This is certainly not the case, since any dephasing through the boundaries of the barrier can contribute to real time (phase-time model). An alternative approach for evaluating this duration is represented by the Feynman's transition elements [8]. By applying these, we can obtain a contribution to the tunneling time of the type $(L/v)\exp(-S_0/\hbar)$; that is, a result according to which the real time duration is provided by the semiclassical time (L/v) attenuated by the factor $\exp(-S_0/\hbar)$, with S_0 as the classical action. In typical cases, with S_0 of the order of a few units of $\hbar$, this factor is of the order of $10^{-2}-10^{-3}$, and a contribution of a few picoseconds is compatible with semiclassical times of the order of 10^2 ps, as in Josephson junctions [6]. In this work, the conclusion has been drawn that this contribution should be added to the one evaluated by the stochastic model which, in this case, turned out to be on the same scale of picoseconds. On the contrary, in the light of the present work, we have to believe that the two contributions; namely the one given by the stochastic model and the one evaluated by the transition elements, are really the *same* result. In fact, even within the framework of the transition elements, we arrived at a result of the type (see the last paragraph)

$$\langle t\rangle \simeq \frac{imc^2}{\hbar}\left(\frac{L}{v}\right)^2, \qquad (2)$$

which according to the prescription by Gaveau *et al.* [9], i.e., by identifying $imc^2/\hbar$ with a, becomes

$$\langle t\rangle_R \simeq a\left(\frac{L}{v}\right)^2 \qquad (3)$$

exactly as the real part in Eq. (1). This result tends to put the stochastic model in a different light, one that is not limited by the presence of dissipation (always present in macroscopic systems), but that is also capable of interpreting situations in which the dissipation is absent or negligible.

This, however, is not the only result of the present work. Another and perhaps more important result, one that is peculiar to the stochastic approach, was obtained when the range of the variable L/v is not limited to *small* values, but is extended, depending on the parameter a, to *large* values of L/v. In this way, we found that real time as a function of L/v shows the unexpected presence of a peak [see Fig. 1(a)]. This is situated approximately at a value of L, so that the semiclassical time L/v is nearly equal to the *time* a^{-1}. Behind this peak, the curve continues with a nearly quadratic

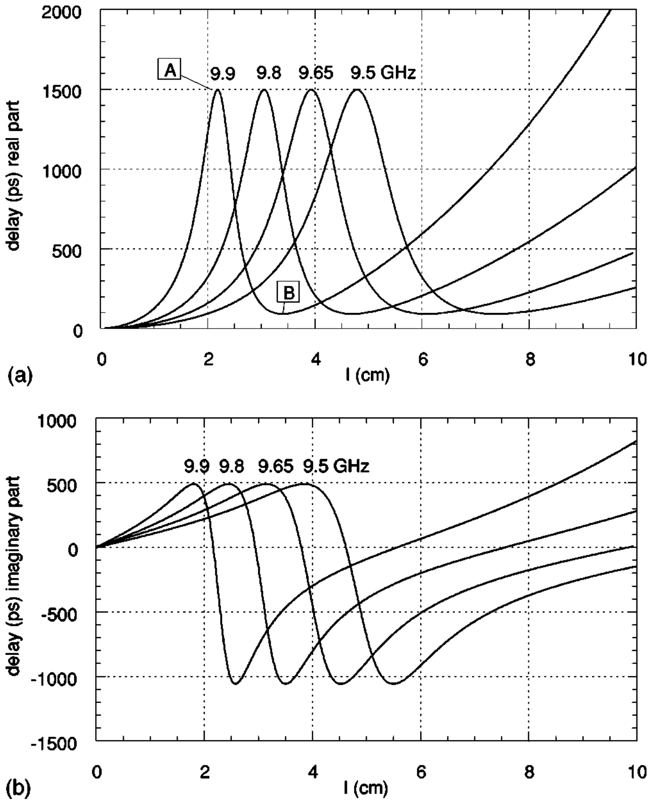

FIG. 1. Real part (a) of the penetration time as a function of the position l inside the barrier (infinitely long) computed for several values of the frequency ν below the cutoff one at 10 GHz and for a fixed value of the dissipative constant $a=2\ (\mathrm{ns})^{-1}$. The position of the peak (whose amplitude is equal to $3/a$ ns) is nearly coincident with $l=v/a$, so by lowering a it tends to increase, while it decreases with increasing a. So, the possibility to detect this peak depends on a suitable choice of the parameter values. A and B indicate the points of stationarity in the curve; (b) same as (a) for the imaginary part.

law. This behavior, i.e., the presence of a peak in the real part of the traversal time, supports the hypothesis of a kind of resonance, which is also confirmed by the curve of the imaginary part of the time. In fact, this curve shows a typical shape [see Fig. 1(b)], with a zero in correspondence with the position of the above-mentioned peak, which is a characteristic of resonances [10]. The origin of this resonance is not completely clear: we note that its occurrence, for $L/v \approx a^{-1}$ could be interpreted by rewriting this condition as $aL \approx v$, and hypothesizing in that it occurs, producing a *strong* increase in real time, when the semiclassical (imaginary) velocity v becomes comparable with the quantity aL, that has velocity dimensions and acts in the sense of slowing down the motion, thus increasing the traversal time of the barrier. More important than the theoretical interpretation is the experimental test of this behavior. Two experiments were performed in the microwave range (wavelength, $\lambda \simeq 3$ cm) and consisted, one in measuring the penetration time in a barrier constituted by a rectangular waveguide excited below the cutoff frequency, the other in measuring the lateral shift of a microwave beam in the case of frustrated total reflection.

In the first experiment the signal (the pulse modulation of the carrier) was taken simultaneously before the input of the barrier (where the waveguide was filled by a dielectric, teflon, so that the frequency of the carrier was *above* the cutoff) and inside the barrier at a variable position l (see the inset in Fig. 2). The two signals were sent to a dual channel oscilloscope (Tektronix TDS 680B) suitable for measuring the temporal delay between the two signals with sufficient accuracy. Reliable measurements required the use of frequencies near the cutoff one ($\nu_0=9.494$ GHz) and a penetration depth in the barrier of a few centimeters, in order to have an acceptable attenuation of the waves. The measurements were performed at several values of the frequency carrier ranging between 9.46 and 9.28 GHz, and were compared with the curve of the real part of the traversal time. This was computed, for a given frequency ν, which deter-

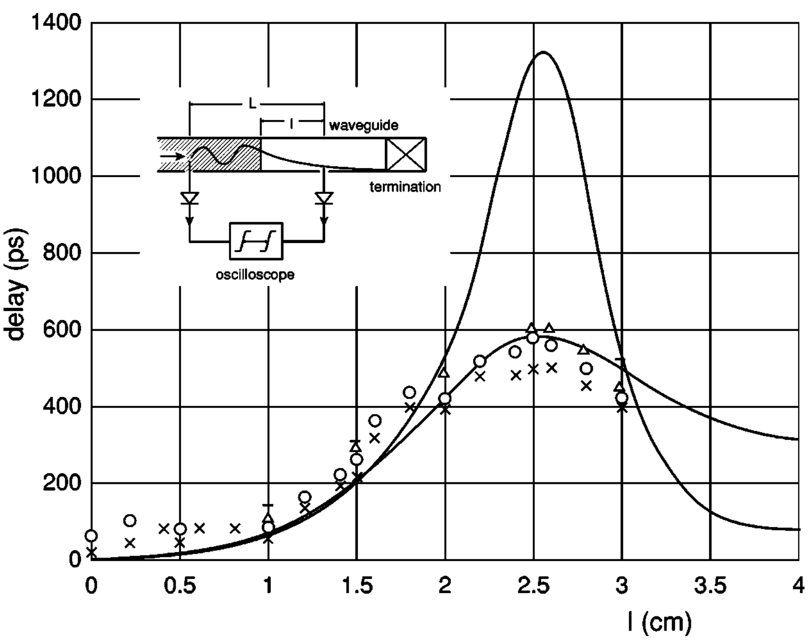

FIG. 2. Penetration time results (small crosses, triangles, and circles refer to different series of measurements) as obtained with the experimental setup shown in the inset. The carrier frequency was $\nu=9.33$ GHz, the cutoff frequency $\nu_0=9.494$ GHz. The upper curve refers to $a=2.25\ (\mathrm{ns})^{-1}$; the lower curve is obtained by including an imaginary part, namely, $a=2.25-i0.5\ (\mathrm{ns})^{-1}$.

mined the velocity through the relation $v=c\sqrt{(\tilde{\nu}_0/\nu)^2-1}$, with $\tilde{\nu}_0$ as the effective cutoff frequency given by $\tilde{\nu}_0=\sqrt{\nu_0^2+a^2\nu^2/c^2}$, by selecting the value for a, which best described the experimental data.

In extreme cases, $\nu=9.46$ and 9.28 GHz, the results obtained showed a clear monotonic (nearly quadratic) behavior, increasing with l, the description of which by the stochastic model required values for a of the order of 1 $(\mathrm{ns})^{-1}$ or less. More interesting were the cases at intermediate frequencies (9.39 and 9.33 GHz), which exhibited more complicated behavior, with the presence of a more or less pronounced peak as predicted by the theoretical model in some cases (see Fig. 1). In Fig. 2 we report the results obtained for $\nu=9.33$ GHz showing a clear increase in the delay up to values of l around 2.5 cm. However, for larger values of l, the delay tends to decrease or, at least, to saturate. The resulting peak was however, not as accentuated as predicted by the above theory, but rather was damped as if a damping coefficient were present in the resonance (a phenomenon analogous to the lowering of the coefficient Q of a resonant circuit or cavity [11]). By taking into account that in tunneling cases, the *dissipative* parameter a represents an imaginary dissipation, i.e., not *dissipative*, but rather *reactive*, it is plausible to introduce a truly dissipative effect by including a suitable imaginary part in parameter a, so that $a\to a+ib$. In this way, we obtained more damped peaks in the curves of the real part of the time, as shown in Fig. 2, where the lower curve is obtained for $a=2.25-i0.5$ $(\mathrm{ns})^{-1}$.

The second experiment consisted of measuring the shift of a beam traversing a gap of a few centimeters between two paraffin prisms while total reflection takes place in the first prism and evanescent waves originate in the gap. This is an extension to the microvave range of an analogous experiment already made in the optical range [12]. With reference to the inset in Fig. 3, for an incidence angle $i=60°$ (the critical angle is 42°) we have quantity D, which is a measure of the traversal time, as twice displacement δ. The latter can be determined by measuring the shift Δs of the beam while the gap is varied from zero to d. Put into a formula, we have

$$D=2\delta=2(d\sin i-\Delta s). \tag{4}$$

Once the shift δ is known, we can determine the traversal time as [12]

$$\tau=\frac{nD}{c\sin i}, \tag{5}$$

where n is the refractive index of the prisms (in our case $n=1.49$) and c is the light velocity in vacuum. In Fig. 3 we report results of τ versus d as they result from the measurements. We note that, in spite of a non-negligible uncertainty [13] in determining of the duration of the process, the data confirm the presence of the peak as predicted by the theoretical model. In Fig. 3, we report the curves as given by the theoretical model: see Eqs. (6) and (7), where the velocity is given, in this case, by $v=c/\sqrt{(\sin i)^2n^2-1}$, [14,15]. The value of a that best fits the experimental data is between 30 and 35 $(\mathrm{ns})^{-1}$. A small imaginary part, say $b=-2.5$ $(\mathrm{ns})^{-1}$, is also admissible.

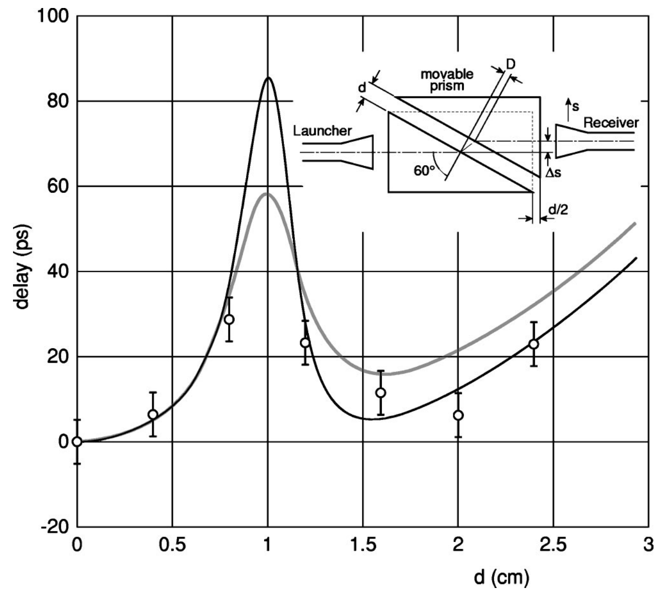

FIG. 3. Traversal time results as a function of the gap width d between two paraffin prisms. The experimental setup is shown in the inset. The microwave beam at 9.33 GHz, in conditions of frustrated total internal reflection, exhibits a shift Δs, in the transmitted part through the gap, which is related to the traversal time. The curves correspond to the real part of the tunneling time as given by the theoretical model for $a=35$ $(\mathrm{ns})^{-1}$ (solid line), and $a=35-i2.5$ $(\mathrm{ns})^{-1}$ (shaded line).

It seems, therefore, that by means of these experiments we have obtained a sufficiently clear demonstration of the validity of the stochastic model for tunneling, even independently of the existence of the peak in the curve of the delay time. This peak, however, strongly supports the theoretical model, even if its implications are rather surprising. In fact, for certain values of l, or d, the traversal time of the forbidden region tends to *decrease*, while the distance *increases*. This unusual behavior deserves further investigation before any definitive conclusion can be safely drawn. It is worth noting, however, that in the region of the resonance peak, the punctual velocity as given by the inverse of the derivative, that is, the limiting values of $\Delta l/\Delta t$ in Figs. 2 and 3, supplies velocities that are infinite in the two stationary points (i.e., A and B in Fig. 1) and negative in the intermediate interval. This behavior is reminiscent of the results recently demonstrated by Wang *et al.* [16], with gain assisted superluminal velocity in light propagation.

Equivalence of the stochastic model with one derived from the transition elements. According to the analysis of Ref. [2], the average of the traversal time of a classically forbidden region can by evaluated as

$$\langle t\rangle=\frac{\int_{-\infty}^{\infty}itg(ir,it)dit}{\int_{-\infty}^{\infty}g(ir,it)dit}, \tag{6}$$

where $g(ir,it)$ is the distribution function of the stochastic processes analytically continued to the imaginary time. When developed, Eq. (6) produces the following result:

$$\langle t\rangle = \frac{r[2\sin ar - ar] + ir[2\cos ar - 1 + 2a^2r^2/3]}{(2\cos ar - 1) + 2i(ar - \sin ar)}$$
$$\equiv \mathrm{Re}\langle t\rangle + i\,\mathrm{Im}\langle t\rangle, \tag{7}$$

where $r = L/v$ is the semiclassical time, and v is the velocity in the forbidden region (barrier). When the argument $r \to 0$, Eq. (7) tends to the simple expression (1). The exact behavior for the real and imaginary part, however, is the one shown in Fig. 1.

We now want to see how it is possible to obtain a result similar to the real part of Eq. (1), namely Eq. (3), within a completely different framework: that of transition elements. On p. 174 of the Feynman and Hibbs book [8], Eq. (7.49) reads

$$\left\langle \left(\frac{\Delta x}{\varepsilon}\right)^2 \right\rangle = -\frac{\hbar}{im\varepsilon}\langle 1\rangle, \tag{8}$$

where ε equals Δt and $\langle 1\rangle$ is the propagator. The quantity $\hbar/m\varepsilon$ has the dimensions of (velocity)^2, the same as $(\Delta x/\varepsilon)^2$. Assuming that the operation of average does not modify the dimensions [17], we obtain the relation

$$\varepsilon\langle 1\rangle = i\frac{m}{\hbar}\langle(\Delta x)^2\rangle, \tag{9}$$

and by multiplying by $c^2 \leftrightarrow v^2$, we obtain

$$\varepsilon\langle 1\rangle = i\frac{mc^2}{\hbar}\left\langle \left(\frac{\Delta x}{v}\right)^2 \right\rangle, \tag{10}$$

which by assuming $\varepsilon\langle 1\rangle = \langle t\rangle_R$ and $\Delta x = L$, becomes Eq. (2); by identifying, as before, $imc^2/\hbar \leftrightarrow a$, it becomes Eq. (3), as previously anticipated. Therefore, for small values of the argument $\Delta x/v$ or L/v, the two approaches give the same result for the real part of the traversal time.

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